THE QUARK-QUARK CORRELATOR: THEORY AND PHENOMENOLOGY

E. DI SALVO

DIPARTIMENTO DI FISICA UNIV. GENOVA

INFN - SEZ. DI GENOVA

COMO, 7-10 SEPTEMBER 2005

SUMMARY

1. THREE PROBLEMS IN HIGH ENERGY REACTIONS

- DETERMINING TRANSVERSITY
- AZIMUTHAL ASYMMETRIES
- FUNCTION g_2

A) TRANSVERSITY

$$h_1(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | \bar{\psi}(0) \gamma_5 \gamma^+ \gamma_\perp \cdot S \, \mathcal{L}(\lambda) \, \psi(\lambda) | P, S \rangle$$

$$\mathcal{L}(\lambda) = \mathbb{P} \exp \left\{ -ig \int_0^\lambda A^+(\lambda') \, d\lambda' \right\}$$

AXIAL GAUGE $\Rightarrow \mathcal{L}(\lambda) = 1$ $\qquad h_1(x) = q^\uparrow(x) - q^\downarrow(x)$

$h_1(x)$ CHIRAL ODD $\Longrightarrow$ ~~DIS~~

"DIRECT" METHODS:

$$\begin{cases} p^\uparrow p^\uparrow \to \mu^+ \mu^- X, \; J/\psi \, X & \sigma \propto h_1 \otimes \bar{h}_1 \\ p^\uparrow p \to \Xi^\uparrow X & \sigma \propto h_1^p \otimes h_1^\Xi \end{cases}$$

"INDIRECT" METHODS: $\quad \ell p^\uparrow \to \ell \pi X \qquad \sigma \propto h_{1T} \otimes H_1^\perp$

$\qquad \ell p^\uparrow \to \ell \pi \pi X \qquad \sigma \propto h_{1T} \otimes (\text{I.F.})$

$h_1(x) = \int d^2 p_\perp \, h_{1T}(x, \vec{p}_\perp^{\,2})$ $\qquad H_1^\perp, \; (\text{I.F.})$: T-ODD

B) AZIMUTHAL ASYMMETRIES

- $\ell\, p^{\uparrow} \longrightarrow \ell\, \pi\, X$ HERMES, COMPASS, CLAS
- $p\, p(\pi) \longrightarrow \mu^+ \mu^- X$ (cos 2φ ASYMM.) NA10
- $p\, N \longrightarrow Y^{\uparrow} X$ (Y = HYPERON)
  $p\, p^{\uparrow} \longrightarrow \pi^0 X$ RHIC

SUCH ASYMMETRIES CANNOT BE EXPLAINED BY pQCD

POSSIBLE EXPLANATIONS:

- QUARK-QUARK-GLUON CORRELATIONS

J. Qiu & G. Sterman: P.R.L. <u>67</u> (1991) 2264; Nucl. Phys. B <u>378</u> (1992) 52

- TRANSVERSE MOMENTUM → T-ODD FUNCTIONS

D. W. Sivers: P. R. D <u>41</u> (1990) 83; P.R. D <u>43</u> (1991) 261

D. Boer, R. Jakob and P.J. Mulders: Nuc. Phys. B <u>564</u> (2000) 471

M. Anselmino et al.: P. R. D <u>71</u> (2005) 074006

J. C. Collins: Ph. Lett. B <u>536</u> (2002) 43

DISCRETE AGREEMENT WITH DRELL-YAN

BUT NOT REPRODUCED $M_{eff.}$ DEPENDENCE

c) FUNCTION $g_2(x)$

A PUZZLING OBJECT

WANDZURA - WILCZEK APPROXIMATION:

$$g_1(x) + g_2(x) = \int_x^1 \frac{dy}{y}\, g_1(y)$$

OPERATOR PRODUCT EXPANSION $\Rightarrow$

$$g_2(x) = g_2^{(2)}(x) + g_2^{(3)}(x)$$

(2), (3) $\to$ TWIST

$$\int_0^1 dx\, x^{n-1} \left\{ g_1(x) + \frac{n}{n-1}\, g_2^{(2)}(x) \right\} = 0$$

$n \geq 3$:

$$\int_0^1 dx\, x^{n-1} \left\{ g_1(x) + \frac{n}{n-1}\, g_2^{(3)}(x) \right\} = \frac{1}{2} \sum_i S_i\, d_n^i\, E_{q,i}^n$$

A PUZZLE:

OPE $\Longrightarrow$ $g_2^{(3)}(x) + g_1(x) \neq 0$

INSTEAD Jaffe & Ji : P.R.L. 67 (1991) 552; N.P.B 375 (1992) 527

$$\Longrightarrow \quad g_2(x) + g_1(x) = (\text{TWIST } 3)$$

2. DEFINITION OF THE CORRELATOR

REACTIONS AT HIGH MOMENTUM TRANSFER:

$$d\sigma \propto (\text{Soft}) \otimes (\text{Hard})$$

FACTORIZATION THEOREMS:

J. C. Collins: Phys. Rev. D 57 (1998) 3051

J. C. Collins & D.E. Soper: Nucl. Phys. B 193 (1981) 381;

Nucl. Phys. B 197 (1982) 446

X. Ji, J. P. Ma & F. Yuan: Phys. Lett. B 597 (2004) 299;

Phys. Rev. D 71 (2005) 034005

"SOFT" FUNCTIONS ENCODED IN

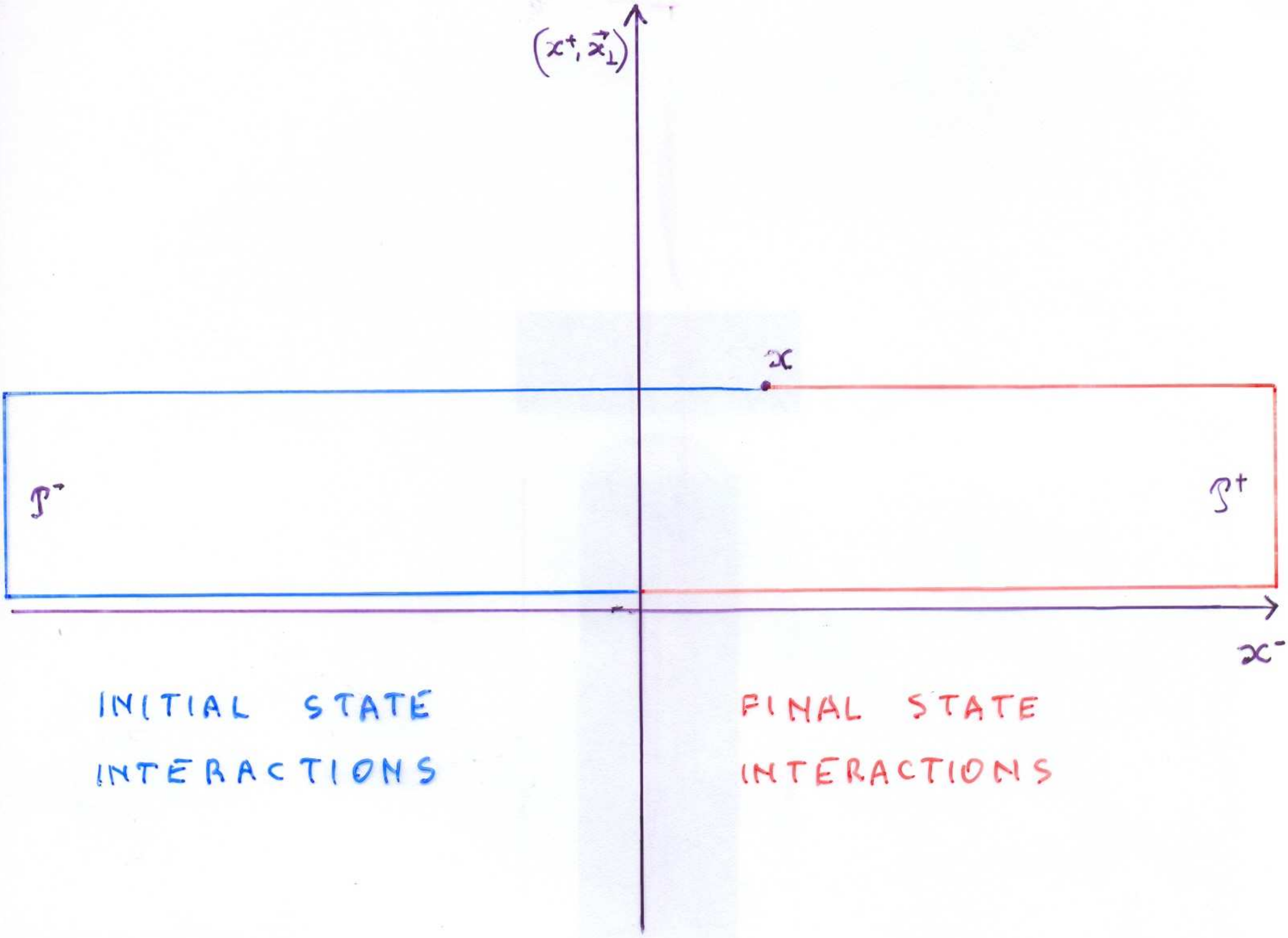
CORRELATOR:

$$\Phi_{ij} = \int \frac{d^4x}{(2\pi)^4} \, e^{ipx} \langle P, S | \bar{\psi}_j(0) \, \mathcal{L}(x) \, \psi_i(x) | P, S \rangle$$

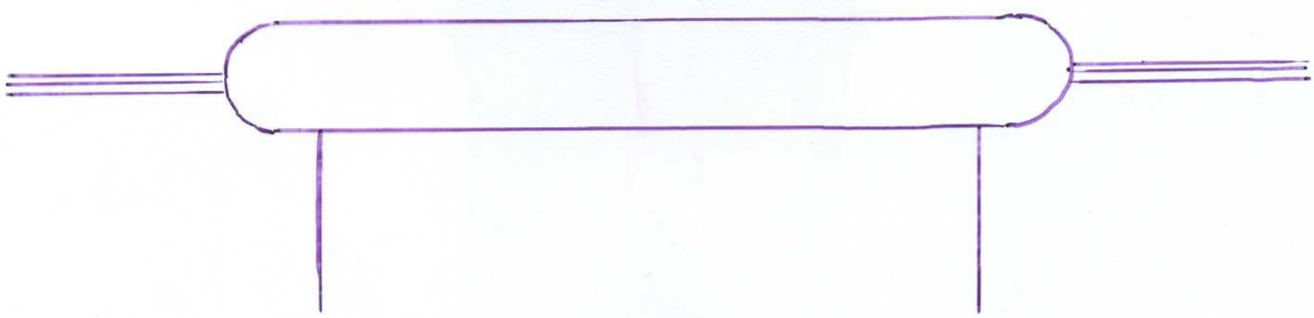
$$\mathcal{L}(x) = \mathbb{P} \exp \left\{ -ig \int_{0\,(\mathcal{P}^{\pm})}^{x} A^{\mu}_a(z) \, \lambda^a \, dz_{\mu} \right\}$$

R. J. Ralston & D.E. Soper: Nucl. Phys. B 152 (1979) 109

P. J. Mulders & R.D. Tangerman: Nucl. Phys. B 461 (1996) 197



$$\Phi_{ij} = \int \frac{d^4x}{(2\pi)^4}\, e^{ipx} \langle P, S|\, \bar{\psi}_j(0)\, \mathcal{L}(x)\, \psi_i(x)\, |P, S\rangle$$



Φ: GREEN FUNCTION : 2 NUCLEON LEGS + 2 QUARK LEGS

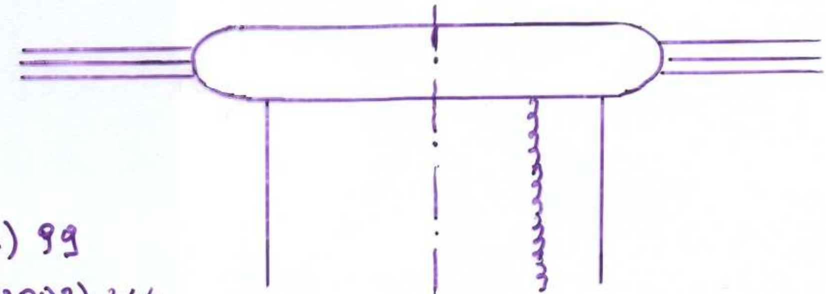
+ 0, 1, 2 ... n ... GLUON LEGS

PROPERTIES OF Φ

- HERMITICITY OF $\gamma_0 \Phi$
- P-INVARIANCE
- BUT T-EVEN AND T-ODD FUNCTIONS

T-ODD FUNCTIONS IN STRONG INTERACTIONS:

INTERFERENCE



S. Brodsky et al.: P.L. B 530 (2002) 99
Nuc. Phys. B 642 (2002) 344

DIRAC ALGEBRA: 16 ELEMENTS

$\downarrow$

32 FUNCTIONS IN ALL:

K. Goeke, A. Metz & M. Schlegel: hep-ph/0504130
→ Ph. Lett. B 618 (2005) 90

DISTRIBUTION FUNCTIONS

PROJECTIONS ONTO DIRAC COMPONENTS :

$$\Phi^{\Gamma} = \frac{1}{2} \int dp^{-} \, \mathrm{tr} \left(\Phi \Gamma \right) \qquad \Gamma \in \text{DIRAC ALG.}$$

$$\Phi^{\gamma^{+}} \supset f_1(x, \vec{p}_\perp^{\,2}) \qquad \text{UNPOLARIZED DENSITY}$$

$$\Phi^{\gamma_5 \gamma^{+}} = g_{1L}(x, \vec{p}_\perp^{\,2}) \qquad \text{LONG. POL. DENSITY}$$

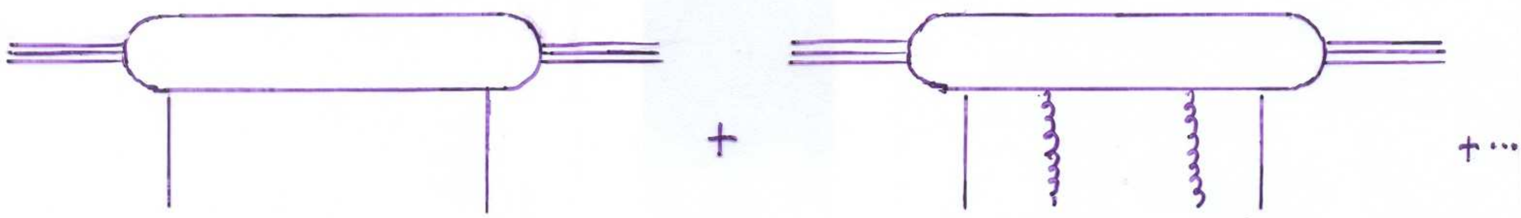
$$\Phi^{\gamma_5 \gamma^{+} \gamma^{i}} \supset h_{1T}(x, \vec{p}_\perp^{\,2}) \qquad \text{TRANSVERSITY}$$

3. SPLITTING OF Φ INTO A T-EVEN AND A T-ODD CONTRIBUTION

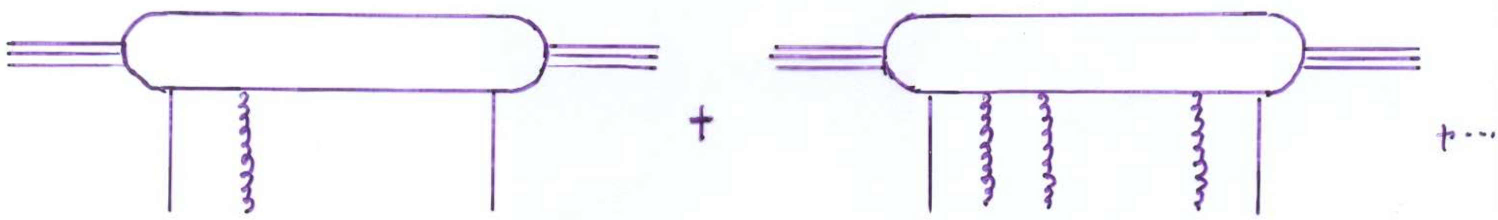
$$\mathcal{L}_E(x) = \mathbb{P} \cos\left\{ g \int_{0(\mathcal{S}^+)}^{x} A_a^\mu(z)\, \lambda^a\, dz_\mu \right\}$$

$$\mathcal{L}_O(x) = -i\, \mathbb{P} \sin\left\{ g \int_{0(\mathcal{S}^+)}^{x} A_a^\mu(z)\, \lambda^a\, dz_\mu \right\}$$

$$\mathcal{L}_E(x) \longrightarrow \Phi_E = \Phi_E^{(0)} + g^2 \Phi_E^{(2)} + g^4 \Phi_E^{(4)} + \cdots :$$ T-EVEN $\mathcal{S}^+ + \mathcal{S}^-$



$$\mathcal{L}_O(x) \longrightarrow \Phi_O = g\Phi_O^{(1)} + g^3 \Phi_O^{(3)} + g^5 \Phi_O^{(5)} + \cdots :$$ T-ODD $\mathcal{S}^+ - \mathcal{S}^-$



"SOFT" FUNCTIONS

- T-EVEN FUNCTIONS: $f_E = \frac{1}{2} \mathrm{tr}\left(\phi_E \Gamma\right) = \phi_E^\Gamma$

IN PARTICULAR:

$$f_1(x, \vec{p}_\perp^{\,2}) = \phi_E^{\gamma^+}$$

$$g_{1L}(x, \vec{p}_\perp^{\,2}) = \phi_E^{\gamma_5\gamma^+}$$

$$h_{1T}(x, \vec{p}_\perp^{\,2}) = \phi_E^{\gamma_5\gamma^+\gamma_\perp\cdot S}$$

- T-ODD FUNCTIONS: $f_O = \frac{1}{2} \mathrm{tr}\left(\phi_O \Gamma\right) = \phi_O^\Gamma$

IN PARTICULAR:

- $$f_{1T}^\perp(x, \vec{p}_\perp^{\,2})\, \varepsilon^{+\alpha\beta\gamma} \frac{p_\perp^\alpha}{\mu} \frac{P^\beta}{|\vec{P}|} S^\gamma = \phi_O^{\gamma^+}$$

UNPOLARIZED QUARK DENSITY IN A TRANSVERSELY POLARIZED NUCLEON (SIVERS FUNC.)

- $$h_1^\perp(x, \vec{p}_\perp^{\,2}) = \phi_O^{\gamma_5\gamma^+\gamma_\perp\cdot\tau_\perp} \qquad \tau_\perp^\alpha = \varepsilon^{+\alpha\beta\gamma} \frac{p_\perp^\beta}{\mu} \frac{P^\gamma}{|\vec{P}|}$$

QUARK TRANSVERSITY IN AN UNPOLARIZED NUCLEON ("COLLINS" FUNCTION)

4. EQUATIONS OF MOTION

POLITZER'S THEOREM

(H. D. Politzer: Nucl. Phys. B 172 (1980) 349

$$\Downarrow$$

$$\langle P,S|\bar{\psi}_j(0)\,\mathcal{L}(x)\,(i\not{D}-m_q)_{il}\,\psi_l(x)|P,S\rangle = 0$$

$$\Downarrow$$

SETTING

$$\Phi_E = \Phi_E^{(0)} + g^2\,\Phi_E^{(2)} + \cdots, \qquad \Phi_O = g\,\Phi_O^{(1)} + \cdots,$$

WE GET

$$(\not{p}-m_q)\,\Phi_E^{(0)}(p) = 0,$$

$$(\not{p}-m_q)\,\Phi_O^{(1)}(p) = \Psi_O^{(1)}(p),$$

$$(\not{p}-m_q)\,\Phi_E^{(2)}(p) = \Psi_E^{(2)}(p),$$

WHERE, IN AN AXIAL GAUGE,

$$\left[\Psi_O^{(1)}(p)\right]_{ij} = \int \frac{d^4x}{(2\pi)^4}\, e^{ipx}\,\langle P,S|\bar{\psi}_j(0)\left[\not{A}_\infty\right]_{ik}\psi_k(x)|P,S\rangle,$$

$$\left[\Psi_E^{(2)}(p)\right]_{ij} = \frac{1}{\not{p}-m_q}\int\frac{d^4x}{(2\pi)^4}\, e^{ipx}\,\langle P,S|\bar{\psi}_i(0)\,\mathbb{P}\{A_\infty^2\}\,\psi_i(x)|P,S\rangle$$

$$A_{\infty\mu} = A_{\perp\mu}(x^- = \infty, x^+, \vec{x}_\perp)$$

A) ZERO ORDER APPROXIMATION

$$(\not{p} - m_q)\, \Phi_E^{(0)} = 0 \qquad + \qquad \text{HERMITICITY OF } \gamma_0 \Phi_E$$

$$\Downarrow$$

$$\Phi_E^{(0)} = \frac{p^+}{2} (\not{p} + m_q) \left(f_1^{(0)} + g_{1L}^{(0)} \not{S}^q_{\parallel} \gamma_5 + h_{1T}^{(0)} \not{S}^q_{\perp} \gamma_5 \right) \delta(p^2 - m_q^2)$$

ONLY 3 INDEPENDENT FUNCTIONS

$$\Downarrow$$

RELATIONS AMONG DIFFERENT DIRAC

COMPONENTS :

TWIST 2

$\boxed{g_{1T}^{(0)} = h_{1T}^{(0)}}$ $\quad g_{1T}$: LONGIT. POL. QUARK DENSITY IN A TRANSV. POL. NUCLEON

IMPORTANT FOR DETERMINING TRANSVERSITY

$\boxed{h_{1L}^{(0)} = g_{1L}^{(0)}}$ $\quad h_{1L}$: TRANSV. POL. QUARK DENSITY IN A LONGIT. POL. NUCLEON

TWIST 3

$\boxed{g_T^{(0)} = m_q\, h_1^{(0)}}$ $\qquad g_T = \int \Phi_E^{\gamma_5 \gamma_\perp \cdot S}\, d^2 p_\perp = g_1 + g_2$

B) FIRST ORDER (T-ODD) CORRECTION

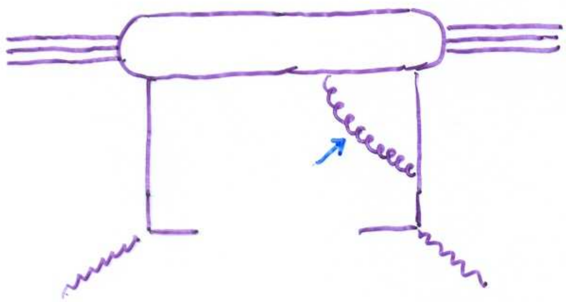
$$(\not{p} - m_q)\, \Phi_0^{(1)} = \Psi_0^{(1)} \qquad \Psi_0^{(1)} \rightarrow$$

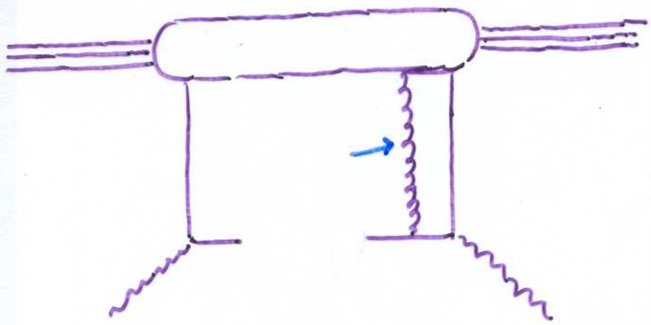
$q\,q\cdot g$ CORRELATOR

$$\Downarrow$$

$$\Phi_0^{(1)} = (\not{p} + m_q)\, \Psi_0^{(1)}\, \mathcal{P}\frac{1}{p^2 - m_q^2} \Rightarrow \boxed{\Phi_0^{(1)} \propto \frac{|\vec{p}_\perp|}{p^+}}$$

EFFECTIVE T-ODD FUNCTIONS:





SOFT GLUON APPROXIMATION ($|\vec{p}_\perp| \ll Q$):

J. C. Collins: P. R. D <u>57</u> (1998) 3051; P.L. B <u>356</u> (2002) 43

T-ODD FUNCTIONS $\propto Q^{-1}$

$$\Downarrow$$

E. G.,

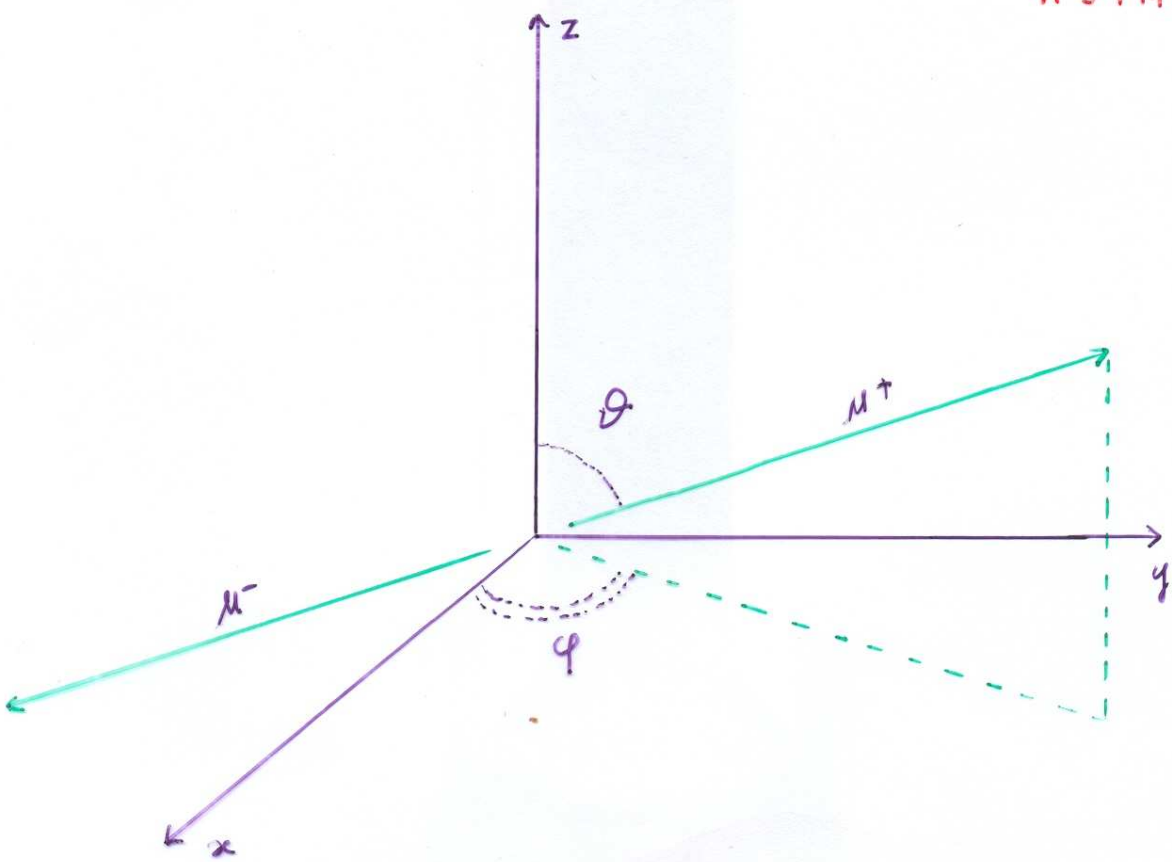
SINGLY POLARIZED SIDIS ASYMM.: $h_{1T} \otimes \underline{\underline{H_1^\perp}} \propto Q^{-1}$

UNPOLARIZED DRELL-YAN ASYMM.: $\underline{\underline{h_1^\perp}} \otimes \underline{\underline{\overline{h}_1^\perp}} \propto Q^{-2}$

UNPOLARIZED DRELL-YAN

$$p\,\pi \to \mu^+ \mu^- X$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1 + \lambda\cos^2\vartheta + \mu\sin 2\vartheta\cos\varphi + \underbrace{\frac{1}{2}\nu\sin^2\vartheta\cos 2\varphi}_{\text{AZIMUTHAL ASYMMETRY}}\right)$$

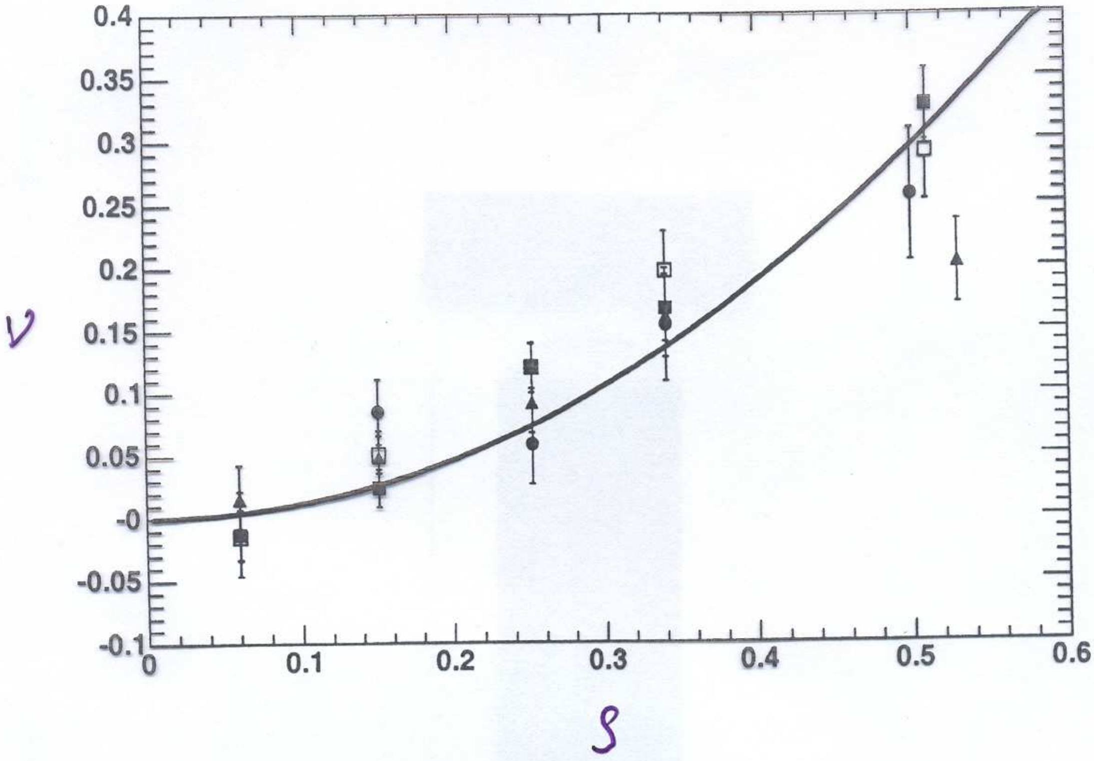


DATA FROM NA 10 COLLAB.:

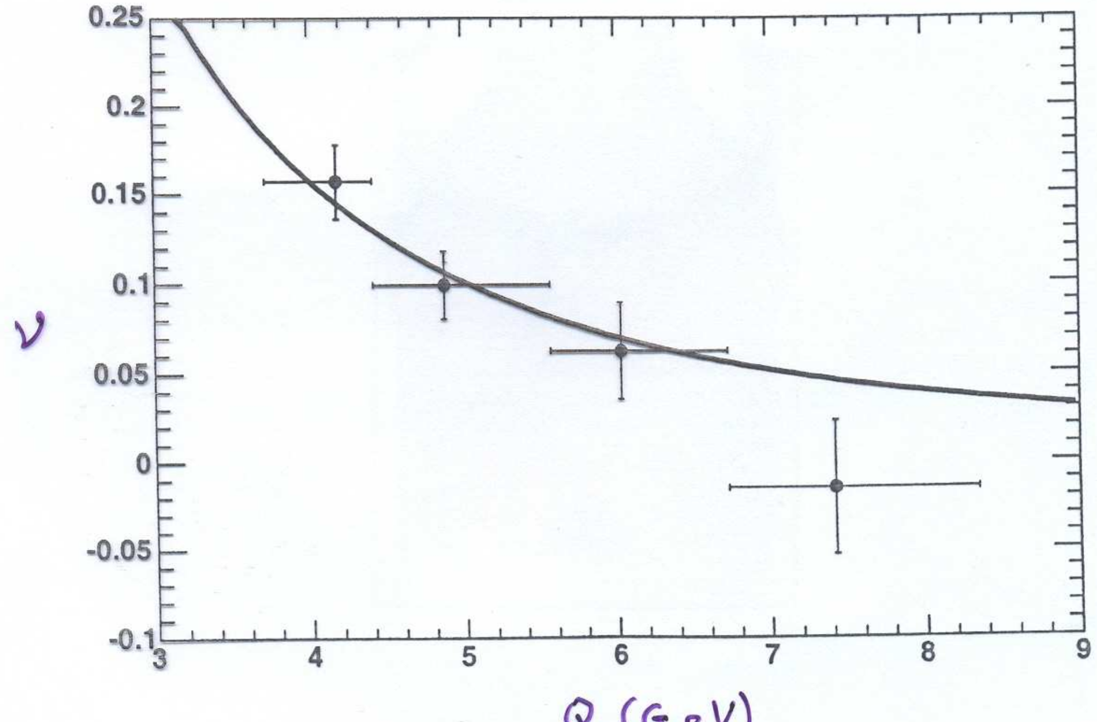
S. Falciano et al.: Z. Phys. C 31 (1986) 513

M. Guanziroli et al.: Z. Phys. C 37 (1988) 545

RESULT EOM: $\boxed{\nu = A\,\dfrac{q_\perp^2}{Q^2}}$ $\quad q_\perp$ = T. M. OF $(\mu^+\mu^-)$

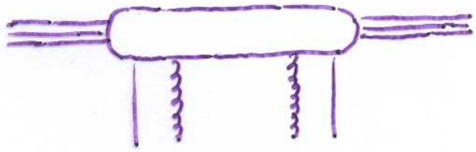


$$\rho = \frac{q_\perp}{Q}$$



c) SECOND ORDER (T-EVEN) CORRECTION

$$(\not p - m_q)\,\Phi_E^{(2)} = \Psi_E^{(2)}$$

$\Psi_E^{(2)} \longrightarrow$ 

qqgg CORRELATOR

$$\Phi_E^{(2)} = (\not p + m_q)\,\Psi_E^{(2)}\,\wp\,\frac{1}{p^2 - m_q^2} \Longrightarrow \boxed{\Phi_E^{(2)} \propto \frac{|\vec p_\perp|^2}{p^{+2}}}$$

TO SUMMARIZE, Φ_E READS AS

$$\Phi_E = \frac{p^+}{2}(\not p + m_q)\left(f_1 + g_{1L}\,\gamma_5\,\not S_\parallel^q + h_{1T}\,\gamma_5\,\not S_\perp^q\right)\delta(p^2 - m_q^2) + g^2(\not p + m_q)\,\Psi_E^{(2)}\,\wp\,\frac{1}{p^2 - m_q^2} + O\left(g^4\,\frac{|\vec p_\perp|^4}{p^{+4}}\right)$$

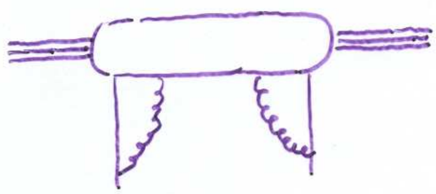
$$\Downarrow$$

$$h_{1T} = g_{1T} + O\left(g^2\,\frac{|\vec p_\perp|^2}{p^{+2}}\right) \longrightarrow O\left(g^2\,\frac{|\vec p_\perp|^2}{Q^2}\right)$$

$$g_T = g_1 + g_2 = m_q\,h_1^{(0)} + (\not p + m_q)\,\frac{1}{2}\,\mathrm{tr}\left(\Psi_E^{(2)}\,\gamma_5\,\gamma_\perp\cdot S\right)\wp\,\frac{1}{p^2 - m_q^2} + O\left(g^4\,\frac{|\vec p_\perp|^4}{p^{+4}}\right)$$

$O\left(g^2\,\frac{|\vec p_\perp|^2}{Q^2}\right) \rightarrow$ PREDICTION

$\Rightarrow$ LIMITS ON OPE FOR g_2

"EFFECTIVE" FUNCTIONS: $\Phi_E \rightarrow \bar\Phi_E$, E.G. 

$$g_T^{eff} = \frac{1}{2}\,\mathrm{tr}\left(\bar\Phi_E\,\gamma_5\,\gamma_\perp\cdot S\right)$$ IS JUST TWIST 3

$\downarrow$

JAFFE - JI

5. CONCLUSIONS

1. APPROXIMATE EQUALITIES BETWEEN

CHIRAL ODD AND CHIRAL EVEN FUNCTIONS:

$$h_{1T}(x, \vec{p}_\perp^{\,2}) = g_{1T}(x, \vec{p}_\perp^{\,2}) + O\left(g^2 \frac{|\vec{p}_\perp|^2}{Q^2}\right) \rightarrow \text{TRANSVERSITY}$$

$$h_{1L}(x, \vec{p}_\perp^{\,2}) = g_{1L}(x, \vec{p}_\perp^{\,2}) + O\left(g^2 \frac{|\vec{p}_\perp^{\,2}|}{Q^2}\right)$$

2. AZIMUTHAL ASYMMETRIES:

qqg CORRELATIONS $\propto Q^{-1}$

AGREEMENT WITH UNPOL. A. A. DRELL-YAN:

$$h_1 \otimes \bar{h}_1 \propto Q^{-2}$$

EFFECTIVE T-ODD FUNCTIONS $\rightarrow$ APPROXIMATE SOLUTIONS TO E.O.M. (SOFT GLUON APPROX.)

3. PREDICTIONS ABOUT $g_2(x)$:

$$g_1(x) + g_2(x) = m_q h_1^{(0)}(x) + O\left(g^2 \frac{|\vec{p}_\perp|^2}{Q^2}\right)$$

$\Longrightarrow$ LIMITS ON OPE FOR TRANSV. POL. DIS

"EFFECTIVE" $g_1 + g_2$ JUST TWIST 3